Proceedings of the 4th Conference of Mathematical Society of Moldova
CMSM4’2017, June 25-July 2, 2017, Chișinău, Republic of Moldova

Wolfram Mathematica as an environment for solving concave network transportation problems

Tatiana Pașa, Valeriu Ungureanu

Abstract

In this work, we consider a transportation problem on a network with concave cost functions and constrained flows on arcs and expose an approach to its solving via Wolfram language algorithm implementation in Wolfram Mathematica System. Our original results are compared with results obtained by applying built-in Wolfram Language functions on a family of test problems.

Keywords: network transportation problem, optimal solution, concave function.

1 Introduction

The concept of transport network may be used to model various economic processes to obtain minimal cost programs for commodity transportation from sources to destinations knowing the available quantities and demands.

We describe the problem of network transportation for which the quantities of commodity transported through each arc is constrained both from above and bottom, and the costs of transportation associated with arcs are defined by linear-concave functions on arc flows.

2 Problem formulation

Let us consider the network transportation problem described by the graph:

$$= (V, E),\ |V| = n,\ |E| = m.$$

A real function of production and consumption $q = V \to R$ is defined on the finite set of its vertices V. Linear-concave cost functions $\varphi_e(x_e)$ are defined for each arc flows.

We need to determine such a flow x^* that minimizes nonlinear objective function $F(x) = \sum_{e \in E} \varphi_e(x_e)$.

2.1 Standard network

The quantity $p(v_0)$ of commodity available for the source v_0 coincides with the required demand $p(v_t)$ of destination v_t in commodity units.

It is required to solve the nonlinear optimization problem:

$$F(x^*) = \min_{x \in X} F(x),$$

where X is a set of admissible flows on described by the following system:

$$\sum_{e \in E^+(v)} x(e) - \sum_{e \in E^-(v)} x(e) = \begin{cases} -p(v) & v = v_0 \\ 0, & V/\{v_0, v_t\} \\ p(v), & v = v_t \end{cases}$$

with both non-negativity constraints and constraints on the transportation capacities of arcs $l(e) \leq x(e) \leq u(e)$, for all $e \in E$.

2.2 Network with one source and several destinations

The quantity $p(v_0)$ of commodity available for the source v_0 coincides with the required demand $\sum_{v \in V_t} p(v)$ for the destinations V_t in commodity units.

We need to solve the nonlinear optimization problem:

$$F(x^*) = \min_{x \in X} F(x),$$

where X is a set of admissible flows on described by the following system:

$$\sum_{e \in E^+(v)} x(e) - \sum_{e \in E^-(v)} x(e) = \begin{cases} -\sum_{v \in V_t} p(v), & v = v_0 \\ 0, & V/V_t \backslash \{v_0\} \\ p(v), & v \in V_t \end{cases}$$

with both non-negativity constraints and constraints on the transportation capacities of arcs $l(e) \leq x(e) \leq u(e)$, for all $e \in E$.

2.3 Network with several sources and destinations

The quantity $\sum_{v \in V_0} p(v)$ of commodity available for the sources V_0 coincides with the required demand $\sum_{v \in V_t} p(v)$ for the destinations V_t in commodity units.

We need to solve the nonlinear optimization problem:

$$F(x^*) = \min_{x \in X} F(x),$$

where X is a set of admissible flows on G described by the following system:

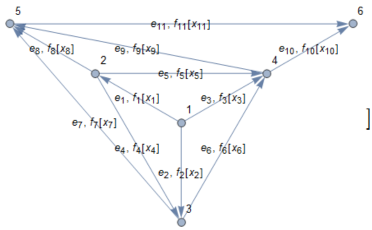
$$\sum_{e\in E^{+}(v)} x(e) - \sum_{e\in E^{-}(v)} x(e) = \begin{cases} -\sum_{v\in V_t} p(v), & v \in V_0 \\ 0, & V/V_t\backslash\{v_0\} \\ \sum_{v\in V_0} p(v), & v \in V_t \end{cases}$$

with both non-negativity constraints and constraints on the transportation capacities of arcs $l(e) \le x(e) \le u(e)$, for all $e \in E$.

3 An algorithm and Wolfram Mathematica program

An original approach to solving the above problems is considered. It may be exposed briefly by the means of the Wolfram language:

```
In[109]:= A = IncidenceMatrix[
```




```
];

{n, m} = Dimensions@A;
b = Table[{0, 0}, n]; b⟦1, 1⟧ = -10; b⟦n, 1⟧ = 10;
X = Array[x, m];
l = {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1};
u = {12, 12, 14, 14, 14, 14, 12, 14, 12, 14, 14};
lu = Table[{l⟦i⟧, u⟦i⟧}, {i, m}];

γ = {1, 1, 2, 2, 2, 2, 1, 2, 1, 2, 2};
δ = {1, 1, 2, 2, 2, 2, 1, 2, 1, 2, 2};
                   ⎧ γ⟦j⟧ x        x ≤ δ⟦j⟧
f[j_Integer, x_] := ⎨ γ⟦j⟧ δ⟦j⟧    x > δ⟦j⟧
                   ⎩ 0             True
                    ⎧ f[j, x] / x   x > 0
fd[j_Integer, x_] := ⎨ γ⟦j⟧          x == 0
                    ⎩ 0             True

X1 = X0 = Values@FindInstance[{A.X == b⟦All, 1⟧, X ≥ 0}, X]⟦1⟧;
F0 = Sum[f[j, X0⟦j⟧], {j, 1, m}];
Z1 = -∞;

i = 0;
While[Not[(Z1 > F0) || (Z1 == F0 && X0 == X1)],
 X0 = X1; F0 = Sum[f[j, X0⟦j⟧], {j, 1, m}];
 c = Table[fd[j, X0⟦j⟧], {j, 1, m}];
 X1 = LinearProgramming[c, A, b, lu]; Z1 = Sum[c⟦j⟧ X1⟦j⟧, {j, 1, m}];
 Print[++i];
]
X1
Sum[f[j, X1⟦j⟧], {j, 1, m}]
```

The above code is exposed for a particular example. But it may be used in the same manner to solve any problem.

4 Conclusion

A series of tests were provided on different test problems to verify efficiency of the approach and program. Our original results have been compared with results obtained by applying built-in Wolfram Language functions on a family of test problems. The approach, algorithm and program proved to be more efficient than built-in Wolfram Mathematica System functions which use numerical algorithms. Our approach, algorithm and program give a more fast execution and solving time than built-in Wolfram Mathematica symbolic functions and methods.

References

[1] T. Pașa, V. Ungureanu. *Solving the transportation problem with piecewise-linear concave cost function on edge flows*, A 20-a Conferință a Societății de Probabilități și Statistică din România, Academia Forțelor Aeriene "Henri Coandă", Departamentul de Științe Fundamentale și Management, Brașov, România, 28-29 aprilie 2017, Editura ASF, p.37.

[2] T. Pasha, D. Lozovanu. *An algorithm for solving the transport problem on network with concave cost functions on flow of edges*. Computer Science Journal of Moldova, vol. 10, no 3, Kishinev (2002), pp. 341-347.

Tatiana Pașa[1], Valeriu Ungureanu[2]

[1]State University of Moldova
E-mail: pasa.tatiana@yahoo.com

[2]State University of Moldova
E-mail: v.a.ungureanu@gmail.com