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Mathematical Modelling of Shock Wave Reflection from a Heat-Conducting Wall

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Abstract

In this work, on the basis of the model of the Navier-Stokes equations, a study of the dynamics of the flow, which arises in the process of reflection of a normally incident shock wave from a heat-conducting wall, is carried out. At the weak intensity of the shock wave and moderate jump in the initial temperatures of gas and wall, analytical expressions for the perturbation of gas parameters, describing the main characteristic features of the reflected flow, are obtained. This allows us to evaluate the influence of viscosity, thermal conductivity and other physical effects on the formation of dissipative and ideal non-viscous and non-heat-conducting zones in the region of the arising flow.

Keywords: the forming gas flow, model of the Navier-Stokes equations, small perturbations of parameters, viscous heat-conducting gas.

1 Introduction

The interaction of real gas with heat-conducting barriers, which are widely encountered in nature and everyday practice, leads to the development of non-stationary flows with a complex internal structure [1]. The process of flow formation is influenced by thermal conductivity and other physical effects [2], [3]. Taking into account the influence of physical effects is of interest for identifying the most important characteristics of the fields of arising flows in gas-solid contact systems. This makes it necessary to study flows, given the influence of physical effects

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accompanying the interaction of gas flows with various obstacles. The theoretical study of flow formation is based on generally accepted models of continuum mechanics. The system of Navier-Stokes equations is widely used for successfully modelling the laws of conservation of mass, momentum and energy [2].

The structure of the forming gas flow is studied on the example of solving the problem of reflection of a normally incident plane shock wave from a heat-conducting wall. At low intensities of the incident wave, gas-dynamic and thermal processes lead to the formation of a flow with small perturbations of parameters, which is described by the Navier-Stokes equations. It is supposed that the gas temperature behind the incident wave differs little from the initial wall temperature. The wall temperature distribution is modelled by the linear heat conduction equation [2]. Taking into account the smallness of the perturbation of the parameters of the arising flow, the study is carried out on the basis of the system of Navier-Stokes equations, linearized around the values of the parameters in the initial state. Analytical solutions to the problem in a linear formulation are obtained in order to analyze the formation of a continuous flow structure with the formation of dissipative and ideal non-viscous and non-heat-conducting zones.

2 Formulation of the problem and analysis of the solution

At the initial moment of time $t = 0$, a normally incident plane shock wave reaches the surface of an impenetrable, stationary heat-conducting wall, with an initial temperature T_w^0. At $t < 0$, the shock wave moved at a constant speed through a homogeneous resting gas, like in an ideal case. The falling wave is followed by a homogeneous flow of viscous heat-conducting gas, occupying a half-space $x > 0$ at $t = 0$, with constant parameters: speed u^0, pressure p^0, temperature T^0 and speed of sound c^0. Hereinafter, the upper index 0 is used to denote the values of the parameters at the initial moment of time, and the lower index w denotes the parameters of the wall.

Gas-dynamic and thermal processes of gas interaction with the wall at $t > 0$ lead to the emergence of non-stationary flows described by the system of complete Navier-Stokes equations, closed by the equation of state for a perfect gas $p = R\rho T$ (where R is the universal gas constant). The boundary conditions are set at a sufficient distance from the wall surface $x \longrightarrow \pm\infty$ and at the gas-wall contact surface ($x = 0$). As $x \longrightarrow \pm\infty$, the parameters are required to tend to the values given in the initial conditions. On the wall surface $x = 0$, conditions of adhesion and continuity of temperatures and heat flow are imposed.

The dynamics of the arising flow is studied in the case of a shock wave of weak intensity and moderate discontinuities in the initial temperatures of the gas and the wall $T_w^0/T^0 \sim 1$. This leads to the formation of flows with small perturbations of parameters. The solution to the problem is sought using its proximity to the solution at the initial moment of time. Each gas-dynamic parameter is represented as a sum of two values. The first value corresponds to the unperturbed value (given in the initial state). The second value expresses a small non-stationary perturbation, keeping the same notation for it (with index ϵ) as for the parameters themselves. After replacing the parameters by the sum of two quantities and discarding the terms with an order of smallness of perturbations higher than the first, we obtain a linearized system of Navier-Stokes equations for disturbances of gas parameters [2]:

$$\begin{gathered} \frac{\partial \rho_\epsilon}{\partial t} + \rho^0 \frac{\partial u_\epsilon}{\partial t} = 0 \\ \rho^0 \frac{\partial u_\epsilon}{\partial t} + \frac{\partial p_\epsilon}{\partial x} = \mu^0 \frac{\partial^2 u_\epsilon}{\partial x^2} \\ \rho^0 c_p \frac{\partial T_\epsilon}{\partial t} = \frac{\partial p_\epsilon}{\partial t} + k^0 \frac{\partial^2 T_\epsilon}{\partial x^2} \\ p_\epsilon = R\rho^0 T_\epsilon + RT^0 \rho_\epsilon \end{gathered} \quad . \tag{1}$$

Here μ is the viscosity coefficient, k is the thermal conductivity coefficient, c_p is the specific heat capacity at constant pressure.

Equations (1) and the equation for the wall temperature distribution form a system of partial differential equations with constant coef-

ficients, which must be solved under appropriate initial and boundary conditions. An analytical solution can be obtained using the Laplace integral transform method with respect to the real variable t. In particular, the distribution of gas and wall temperatures is described by the expressions:

$$T(t,x) = T^0 + T_\epsilon(t,x), \tag{2}$$

$$T_w(t,x) = T_w^0 + T_{w\epsilon}(t,x). \tag{3}$$

Here, small temperature perturbations $T_{w\epsilon}(t,x)$ and $T_\epsilon(t,x)$ are determined from the analytical solution of the linearized problem. Expressions for other gas parameters can be found similarly [2].

The obtained analytical expressions (2), (3) for temperature and other gas parameters allow us to establish the main characteristic features of the emerging gas flow and the wall temperature distribution when a shock wave is reflected from a heat-conducting cold or hot wall. In this case, we can clearly trace the dynamics of the flow field structure development, as well as evaluate the influence of thermal conductivity and other dissipative factors. Based on the obtained analytical solutions, asymptotic expressions for the distribution of gas and wall temperatures for the most intense period of interaction at small times ($t << 1$) were found.

The obtained linearized expressions (2), (3) for the perturbation of temperature and other parameters can be considered as the first approximation of the perturbation of the parameters. Based on them, similarly, one can obtain the following approximation for the parameters of the forming one-dimensional gas flow.

3 Conclusion

On the example of solving the problem of reflection of a normally incident plane shock wave from a heat-conducting wall, an approximate method approach to the solution using the system of Navier-Stokes equations in the one-dimensional case is developed. For a weak intensity of the incident shock wave and a moderate jump in the initial

temperatures of the gas and wall, analytical expressions are obtained for the perturbation of the gas parameters and wall temperature. The obtained linearized solution allows us to establish the main characteristic features of the formation of a continuous structure in the flow field of a viscous heat-conducting gas. In this case, it is possible to evaluate the influence of viscosity, thermal conductivity, and other physical factors on the formation of dissipative and ideal non-viscous and non-thermal conductive zones in the field of the forming gas flow. For the most intense period of interaction at small times, asymptotic expressions for the gas parameters and wall temperature are obtained.

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