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# Experimental Closed-Loop Identification of the Third Order Inertial Systems with Time Delay

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**Abstract**

In this paper, it is analysed the problem of mathematical identification in the closed-loop and it is proposed an approach for experimental identification, based on the undamped step response of the closed-loop control system. According to the proposed procedure, the dynamics of the control object is proposed to be approximated with transfer function with inertia third order and time delay. The obtained results are verified by computer simulation in MATLAB software package.

**Keywords:** Experimental closed-loop identification, mathematical modelling, inertial systems, transfer function, time delay.

## 1 Introduction

Mathematical modelling of a control object in control engineering involves obtaining a mathematical representation of a physical process to describe its behaviour in response to various inputs [6]-[7]. This process is essential for designing effective control strategies and begins with a thorough understanding and description of the system. Identifying the mathematical model that approximates the dynamic of the control object can lead to a qualitative understanding of the system's behavior [1].

System identification can be realized in the open-loop system and closed-loop feedback system [4]. Open-loop identification methods are simple in use and these methods are applied, if the process dynamics

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can be linearized around a nominal operating point, and the process is characterised by the non-varied in time parameters [2]. But for case of using the auto-tuning controllers, it is necessary to obtain the mathematical model of the control object in the feedback closed-loop system, which involves using the closed-loop identification methods [3].

In this paper, it is proposed the procedure for closed-loop identification of the third order inertial system with time delay based on the undamped transient response of the closed-loop system.

## 2 Algorithm of closed-loop identification of the undamped third order inertial system with time delay

The block scheme of the automatic control system is presented in Fig. 1.

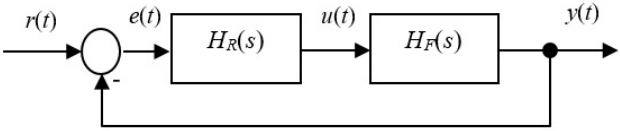



Figure 1. Structural scheme of the automatic control system

It is proposed, that the control object to be approximated with the following transfer function:

$$H_F(s) = \frac{ke^{-\tau s}}{(T_1+1)(T_2+1)(T_3+1)} = \frac{ke^{-\tau s}}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \quad (1)$$

where $k$ - transfer coefficient, $\tau$ - time delay, $T_1, T_2, T_3$ - time constants, $a_0 = T_1T_2T_3$, $a_1 = T_1T_2 + T_1T_3 + T_2T_3$, $a_2 = T_1 + T_2 + T_3$, $a_3 = 1$.

For model estimation, it is proposed to be used P controller with transfer function:

$$H_R(s) = k_p, \quad (2)$$

where $k_p$ is a proportional tuning parameter.

From the transfer function (1), the unknown coefficients $a_0, a_1, a_2, a_3$ are proposed to be calculated based on the values of the $k_{cr}$ – proportional critic transfer coefficient of the P controller, when system achieves undamped step response and $T_{cr}$ – the period of oscillations. To achieve this state of the system, it is considered to be given the closed-loop system with P controller (Fig. 1) and the proportional tuning parameter $k_p > 0$ is varied, until the system achieves the undamped step response presented in Fig. 2.

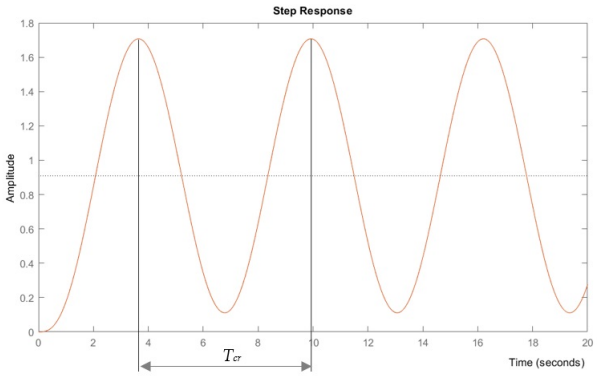



Figure 2. Undamped step response of the closed-loop system

If the values of period oscillations $T_{cr}$ are known, then the value of natural frequency is calculated by [5]:

$$\omega_n = \frac{2\pi}{T_{cr}}. \tag{3}$$

The closed-loop characteristic equation of the system with P controller is the following:

$$A(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3 + k k_{cr} e^{-\tau s}. \tag{4}$$

It is proposed the following substitution of the Laplace transform:

$$s = \omega_n j. \tag{5}$$

Based on Eq. (5), the characteristic equation (4) will become:

$$A(\omega_n j) = (-a_1\omega_n^2 + a_3 + k_{cr}kcos(\tau\omega)) + j(-a_0\omega_n^3 + a_2\omega - kk_{cr}sin(\tau\omega)) =$$

$$= P(\omega) + jQ(\omega). \tag{6}$$

Next, it was set the real and imaginary part with zero, denoted as $P(\omega) = 0$ and $Q(\omega) = 0$; based on this equaling, there are obtained the following expressions for calculation the model parameters $a_1$, $a_2$:

$$a_1 = \frac{a_3 + k_{cr}kcos(\tau\omega_n)}{\omega_n^2}; a_2 = \frac{a_0\omega_n^3 + kk_{cr}sin(\tau\omega_n)}{\omega_n}; a_3 = 1. \tag{7}$$

The coefficient $a_0$ remains unknown and for the calculation of this coefficient it is used the maximum stability degree criterion [7]. According to this criterion, the maximum displacement of the dominant poles of closed loop system to the imaginary axe in the left complex half plane is denoted by the $\eta_{max}$ that is equal to:

$$\eta_{max} = \frac{a_1}{(n+1)a_0}, \tag{8}$$

where $n$ is order of the characteristic equation of the closed-loop system.

And it is known that the maximum displacement is known as maximum stability degree of the system, denoted by $J$.

For the case, when steady state error will be $\epsilon_{st} = 0.02$, the settling time is equal to:

$$t_s \approx \frac{4}{J}. \tag{9}$$

By the variation of the value of proportional tuning parameter $k_p > 0$, it is possible to achieve the critically damped step response. And according to this response, there can be calculated the value of settling time -$t_s$. If this value is known, based on Eq.(8)-(9), it is obtained an expression for calculation $a_0$:

$$a_0 = \frac{a_1 t_s}{20}. \tag{10}$$

# 3 Study Case and Computer Simulation

For the verification of the proposed algorithm of experimental identification, it was supposed that the control object to be described by the following tranfer function:

$$H_F(s) = \frac{e^{-20s}}{150s^3 + 95s^2 + 18s + 1}. \tag{11}$$

The given system (11) was simulated in MATLAB with P controller, where the proportional tuning parameter $k_p$ was varied until it was achieved the undamped transient response of the closed loop system for $k_{cr} = 1.53$.

The obtained undamped step response is presented in Fig. 3, from which there are calculated the period of oscillation $T_{cr} = 70.95s$. The value of natural frequency was determinated by the expresion (3): $\omega_n = 0.0885$.

Next, according to Eq.(7), it was calculated the $a_1$ parameter of the identified transfer function: $a_1 = 88.774$.

Further, the proportional tuning parameter $k_p > 0$ was varied till it achieved the critically damped step response. And according to this response, it was calculated the value of settling time $t_s = 35$ sec. If this value is known, based on Eq.(10), it was calculated $a_0$:

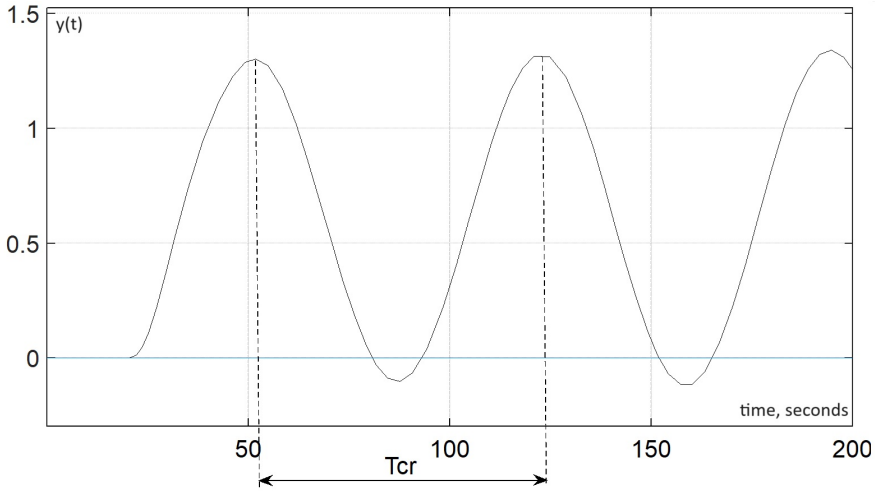



Figure 3. Underdamped step response of the closed-loop system

$$a_0 = \frac{a_1 t_s}{20} = \frac{88.774 * 35}{20} = 155.355. \tag{12}$$

If the value of $a_0$ is known, then it can be calculated the value of $a_2$:

$$a_2 = \frac{a_0\omega_n^3 + kk_{cr}sin(\tau\omega_n)}{\omega_n} = 18.1695. \tag{13}$$

According to these calculations, the identified transfer function is the following:

$$H_F(s) = \frac{e^{-20s}}{156.16s^3 + 88.774s^2 + 18.169s + 1}. \tag{14}$$

In Fig. 4, it is presented the comparation between original step response of the open loop system described by the transfer function (11) – curve 1 and the identified transfer function (14) – curve 2.

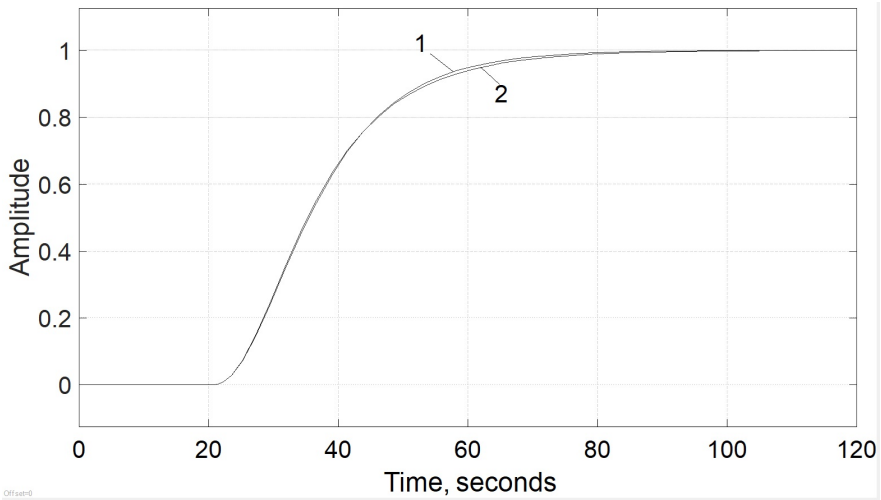



Figure 4. Comparation of the system step responses in the open loop: 1-original transfer function (11); 2-identified transfer function (14)

From Fig. 4, it can be observed that the proposed methodology of model estimation in the closed-loop offers so good results in model estimation.

# 4 Conclusion

In this paper, it was proposed an approach for experimental identification in the closed-loop of the third order inertial system with time delay. The proposed algorithm supposes to be achieved the limit of stability of the closed-loop system, and based on the parameters, that are obtained from the undamped response of the system, there are presented some simple expressions for calculation the parameters of the model. The closed-loop identification was verified by computer simulation and the obtained results demonstrated that the identification procedure in the closed-loop offers good results in process of estimation the mathematical model in the closed-loop, and this method can

be used to design the auto-tuning controllers.

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