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# Semi-Markovian Networks

Iulia Damian

**Abstract**

In stochastic networks, a series of new problems appear, because the output flow of messages from one node forms the input flow to another node (or other nodes), which is not Poisson, and the serving of messages in the nodes of these networks is no longer exponential type. Such networks are called semi-Markovian networks and, therefore, the mathematical analysis of semi-Markovian networks is complicated and requires the application of some modern departments of mathematics.

**Keywords:** queuing system, networks, exponential distribution, semi-Markovian networks, averaging method, diffusion approximation method.

## 1 Introduction

The theory of semimarkovian random evolution has a wide and diversified application, for example, in the theory of inventory, consumption, and transport; in the theory of stochastic differential equations; in the analysis of real stochastic systems that appear when solving problems applied in physics, biology, ecology, and economics. The theory of stochastic networks emerged from the need to exploit informational and numerical systems, economic networks, and telecommunication networks. The processes, which describe the activity of these stochastic networks, can have a rather complex and large structure. The analysis and optimization of complex systems are multiparametric problems, being an important step in solving optimization problems, for complex systems under conditions of uncertainty and the risk of the primary

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mathematical model are usually stochastic. This, for example, appears in the theory of information processing networks. The significant development of the theory of the evolution of stochastic systems is described in the works of Korolyuk V. S. [1,2], Korolyuk V. S. and Svischuka A. V. [3], Korolyuk V. S. and Korolyuk V. V. [4], Korolyuk V. S. and Turbin A.F., Skorokhod A.V., Ghihman I. I. and Skorokhod A.V., and others. The vertiginous development of networks, including wireless information transport networks, have presented a series of pressing problems. Among these problems, there is the development and analysis of mathematical models of these processes that take place in real contemporary systems and networks. In recent years, in the attention of specialists in the field of operational research and applied probabilities, there are the so-called stochastic serving networks. Here a series of new problems arise because in stochastic networks, the output flow of messages from one node forms the input flow to another node (or other nodes), which is not of Poisson type. At the same time, the serving of messages in the nodes of these networks is no longer governed by the exponential distribution. Such networks are called semi-Markov or semi-Markovian networks. That is why the mathematical analysis of semi-Markovian networks is complicated and requires the application of some modern departments of mathematics. Most queuing systems in real life are usually networks. An example is the packet-switched communication system, road networks, etc. However, a queuing network system can be viewed as a simple collection of multiple strings with a single node. Sometimes a queuing network is decomposed into multiple single-node strings as an approximation for analysis purposes. This is due to the fact that, analyzing waiting networks or even tandem networks, they could be very involved, except for simple unrealistic cases. Korolyuk V. S. was concerned with waiting networks. Thus, waiting networks were also investigated in Moldova by Gutuleac E. [5].

## 2 Queuing systems

A queuing system is the system in which the elements end up being processed. The terms “element” and “processing” are generic terms.

Examples:

1) An "element" could refer to customers. The elements could be customers arriving at a system, such as a bank, to receive services (to be "processed").

2) Is in the manufacturing process where an item might be a partially completed part that needs to be machined and so is sent to a station where it is properly machined when possible.

3) The transport unit passing through an intersection is an element that needs to be served, and the service is provided by the intersection in the form of turning on the green light to go through it. In general, in a queuing system, there is a physical or virtual location, sometimes moving, where items end up being processed. If the processor is available, the element can be processed immediately, otherwise it must wait, if the processor is busy. Almost all of us have experienced queuing systems directly or indirectly: directly, when we sit and wait in line, or indirectly, through elements of our own standing in a waiting line, such as when a job waiting to be printed in the printer queue's queue space, or a packet belonging to a person waiting at a router node for processing. In any case, we all want the waiting time to be minimal and also not to be returned from the system due to the unavailability of waiting space. Of course, we all know that if this management system can provide "enough" resources to process the items, then the possibilities of delays or denial of service will be small. However, the system manager providing the service is limited in how many resources can be provided because they cost money.

## 3 The queuing system $[SM|M|1|\infty]^N$ and $SM^{\varepsilon}|M|\infty$

The queuing system $[SM|M|1|\infty]^N$ means:

- the input flow is described by a semi-Markov process
- the servicetime is distributed exponentially

- there are $N$ servers connected by a route probability matrix

These queuing networks are considered with a semi-Markov flow.

The queuing system $SM^{\varepsilon}|M|\infty$ means:

- arrival times are the jumps times of a semi-Markov process on the standard phase space $(E, \varepsilon)$

- the service time is exponentially distributed with one server, while the end of service times are the jumps of Markov process when the queuing system is busy.

The most effective mathematical methods for simplifying waiting systems are: the averaging method and the diffusion approximation method. The averaging method allows to obtain the determined simplified model of the waiting system [6,7]. Since the general mediation model is not fully primary, it is necessary to use the diffusion approximation to evaluate the fluctuations of the initial model. Using the diffusion approximation, we can describe the fluctuations of the initial model against the mean. Solving the optimization problem for the simplified mediation model is much easier than optimizing the original model. Minimizing fluctuations, managing initial network parameters is one of the important practical issues.

# 4 Conclusion

The theory of queuing systems is stochastic in nature. Some components of the system, such as the number of serving nodes, the queue waiting to be served, the intensity of the flow, can be both deterministic and random. Economic examples: some companies satisfy the needs of the population, which can be: shops, hospitals, repair shops, laundries, computer clubs, driving licenses, etc. The company generates revenue that satisfies the customer's needs. Profits depend on constant and variable components. The constant component includes the production cost of assets (buildings, equipment), rent, payment for utilities, minimum wage, taxes, etc. The variable component depends on the

number of actual requests that have been fulfilled in a certain period of time.

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Iulia Damian

Private Institution High School "Columna"
E–mail: iuliagriza@yandex.ru
ORCID: https://orcid.org/0009-0006-4823-2590