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Informational Extended Games And Their Applications

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Abstract

In this article, we analyse informational extended games, i.e., games in which the players choose their actions simultaneously, with assumption that they have some information about the future strategies which will be chosen by other players. For all informational extended games of this type we assume that players' payoff functions are common knowledge. Under these assumptions we define the noncooperative informational extended games and analyse Nash equilibrium. As a particular case of the non-cooperative informational extended games, we analyse the class of bimatrix informational extended games and we present an example of game in order to show the possibility to use the information for this class of games.

Keywords: Informational extended games, Nash equilibrium, best response mapping, point-to-set mapping, fixed points.

1 Introduction

Using the general concept of the Informational Extended Games (IEG) for the non-cooperative games and using some models of informational extended games (as a particular case of Howard meta-games, see Howard [1]), we extend the study of noncooperative informational extended games. The informational extension concept for games has on its basis the assumptions that the participants of the game have possibility to send and to receive (or to guess) some information about the chosen strategies of other participants and about their behavior.

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We construct a type of informational extended game with n players, for which we give the explicit definitions. The main problem which was formulated is to define the conditions of the Nash equilibrium existence for this type of informational extended game.

2 Strategic Form Games and Nash Equilibria

In this section, we analyse games in which the players choose their actions simultaneously (without knowledge of other player choices). The game assume that players' payoff functions are common knowledge.

Definition 1. *A strategic form of the game consists of: a finite set of players* $I = \{1, 2, ..., n\}$*, players' action spaces (the sets of strategies), denoted by* X_i*,* $i \in I$*; and players' payoff functions* $H_i : X \to R, i \in I$*, where* $X = X_1 \times ... \times X_n$*. We refer to such a game as the tuple* $\langle I, (X_i)_{i \in I}, (H_i)_{i \in I} \rangle$ *denoted by* Γ*.*

An outcome is an action profile $(x_1, x_2, ..., x_n)$, and the outcome space is $X = \prod_{i \in I} X_i$. The game is common knowledge among the players. One of the most common interpretations of Nash equilibrium (introduced by John Nash in 1950) is that it is a steady state in the sense that no rational player has an incentive to unilaterally deviate from it. Let us consider $x_{-i} \equiv (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n)$ and $(x_{-i}, y_i) \equiv (x_1, x_2, ..., x_{i-1}, y_i, x_{i+1}, ..., x_n)$.

Definition 2. [3] *A Nash equilibrium of the game* Γ *is an action profile* $x^* \in X$ *such that, for every* $i \in I$*:*

$$H_i(x^*) \geq H_i(x^*_{-i}, x_i) \forall x_i \in X_i.$$

In the following, by $F : X \rightrightarrows 2^Y$ we will denote a point-to-set mapping, were 2^Y is the set of all subsets of Y. A fixed point of the point-to-set mapping $F : X \rightrightarrows 2^Y$ is a point $x^* \in X$ such that $x^* \in F(x^*)$.

The graph for the application F is the set: $gr(F) = \{(x, y) \in X \times Y | x \in X, y \in F(x)\}$. This set can contain some points or can be an empty set. Another and sometimes more convenient way of

defining Nash equilibrium is via the best response correspondences $Br_i : \prod_{j \in I \setminus i} X_j \rightrightarrows X_i$ such that

$$Br_i(x_{-i}) = \{x_i \in X_i : H_i(x) \geq H_i(x_{-i}, x_i') for \forall x_i' \in X_i\}. \quad (1)$$

Definition 3. *A Nash equilibrium is an action profile x^* such that $x_i^* \in Br_i(x_{-i}^*)$ for all $i \in I$.*

If the sets X_i are compacts and the functions H_i are continuous, then the best response set (1) for the player i can be represented by: $Br_i(x_{-i}) =$

$$= Arg \max_{x_i \in X_i} H_i(x_{-i}, x_i) = \{x_i \in X_i | H_i(x_{-i}, x_i) = \max_{y_i \in X_i} H_i(x_{-i}, y_i)\}.$$

Given a strategic form of the game $\Gamma \equiv \langle I, (X_i)i \in I, (H_i)i \in I \rangle$, the set of Nash equilibria is denoted by $NE(\Gamma)$.

Using the best response sets of the players, we consider the point-to-set mapping $Br : \prod_{i \in I} X_i \rightrightarrows 2^X$ such that $Br = (Br_1, Br_2, ..., Br_n)$.

Then we can easily prove that $x^* \in NE(\Gamma) \leftrightarrows x^*$ is a fixed point of the set-valued mapping Br, i.e. $x^* \in Br(x^*)$.

3 Nash equilibria in the noncooperative informational extended games with n players

We analyse a static game with n players:

$$\Gamma = \langle I, X_i, (i = 1, ..., n), H_i, (i = 1, ..., n) \rangle, \quad (2)$$

where $I = \{1, 2, ..., n\}$ is the set of players, the set of strategies for the i-th player is denoted by $X_i, (i = 1, ..., n)$, and the payoff functions are defined by: $H_i : \prod_{i \in I} X_i \to R, (i = 1, ..., n)$.

Next we analyse a static informational extended game with n players. In this informational extended game, we consider that each player is informed of the strategies of the other players which will be chosen. In this case, the sets of the strategies for each player are sets of

functions defined on the product of the sets of strategies of the rest players from the initial game (2). The game is realized as follows: the strategies are chosen simultaneously by players (with assumption that each of them knows which strategies will be chosen by all other players), after that each of players determines his payoff and the game is over. This informational extended game can be described in the normal form by: ${}_n\Gamma = \langle I, \overline{X}_i, i = \overline{1,n}, \overline{H}_i, i = \overline{1,n} \rangle$, where the sets of strategies for players are defined by: $\overline{X}_i = \{\phi_i : \prod_{j \in I, j \neq i} X_j \to X_i\}, i = \overline{1,n}$, where $\prod_{j \in I, j \neq i}, X_j = X_1 \times ... \times X_{i-1} \times X_{i+1} \times ... \times X_n$. The payoff functions are defined on the product of the extended sets of strategies: $\overline{H}_i : \prod_{i \in I} \overline{X}_i \to R, (i = \overline{1,n})$.

For this informational extended game, we have the conditions of Nash equilibrium (see Novac, [5]). We apply the fixed point theorem to prove the following theorem of the Nash equilibrium existence for the informational extended game ${}_n\Gamma$ with n players.

Theorem 1. *[5] Let us consider that for the game ${}_n\Gamma$ the next conditions hold: 1) the sets $X_i \neq \emptyset, (i = \overline{1,n})$ are compacta of Banach spaces, 2) the sets of functions $\overline{X}_i \subset C(\prod_{j \in I, j \neq i} X_j, X_i), i = \overline{1,n}$ are uniformly bounded and the functions from the sets $\overline{X}_i$ are equicontinuous; 3) the payoff functions $H_i(.), i = \overline{1,n}$ are continuous on the compactum $\prod_{i \in I} X_i$ and the functions $\overline{H}_i(.), i = \overline{1,n}$ are concave on $\overline{X}_i$ for $\forall \phi_{-i}$, respectively. Then $NE({}_n\Gamma) \neq \emptyset$.*

4 Applications of the bimatrix informational extended games – Entering a new market

One of the most common interpretations of the "informational extended set of strategies" can be described by the next example. Let us consider an informational extended game with two players. A trivial example is the decision about entering a new market. Suppose the company, which acts as a monopoly in any market, and another outsider company which have to decide on joining or not joining the market. If in a static game, the outsider company would have some information about the reaction of the company, which acts as a monopolist, then it could chose a

better strategy in order to avoid some unexpected situation and not to make his partner to act aggressively. Next we give the mathematical model of the game. Let us consider the bi-matrix game in the normal form: $\Gamma = \langle N, X, Y, A, B \rangle$, where N is the set of players: $N = \{1 = outsidercompany, 2 = monopolycompany\}$, $X = \{1, 2, ..., m\}$ and $Y = \{1, 2, ..., n\}$ – are the sets of strategies for players, A and B are the payoff matrices for players: $A = \{a_{ij}\}, B = \{b_{ij}\}, (i \in X, j \in Y)$. Suppose the sets of strategies are:
- for the Outsider company: $X =$ { *1= enter the market in partnership, 2= enter the market in competition* },
- for the Monopoly company: $Y =$ { *1= acceptance fair competition (loyalty), 2= acceptance partnership, 3= not accept competition, 4= not accept partnership* }.

We can give a dynamical representation of this game. The tree representation of the game in this case can be given as in the picture below (Fig.1).

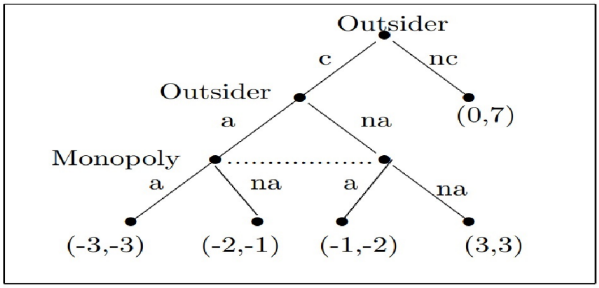


Figure 1. The game "Entering a new market"

If the Outsider company will know previously the strategy that will be applied by the Monopoly company, then he can chose the correct strategy in order to avoid some unexpected situation. So we can define this situation as a bi-matrix informational extended game.

5 Conclusion

The results explained above show that if the amount of the information increases for all participants, then the Nash equilibria sets are larger.

This treatment confirms the importance of the information' possession in all circumstances in the case of the make-decision problems and assure the best result. For the games with two players and discreet sets of strategies, we can define the bi-matrix informational extended games (see Kukushkin and Morozov, [2]) for which it is proved that the set of Nash equilibria is nonempty (see Novac, [4]). The theorem about the conditions of single Nash equilibrium existence for the bi-matrix informational extended games was stated and proved in (Novac, [4]). Our aim is to indicate the importance of the information' possession for all make-decision problems.

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