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Transportation Problems, the Braess Paradox and Network Coordination

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Abstract

Traveling through the transport network or sending information packets via the Internet is implicitly based on game-theoretic considerations: a specific decision-maker (DM), choosing his or her route, takes into account the probability of congestion depending on all DMs, that is, other routes. Based on similar considerations, it is possible to develop models for network traffic. These models explain some paradoxical observations where increasing the capacity of a given network can slow down its traffic under certain circumstances.

Keywords: directed graph, route, subroute, travel time, social optimum, Nash equilibrium.

1 Introduction

Let us develop a model of the transport network and investigate how it reacts to traffic jams [1]. Let us represent the transport network as a directed graph, where the edges are highways, and the nodes are exits, where the DM can leave a certain highway and enter another. Let us assume that each DM wants to travel from node A to node B [2]. It should be taken into account the large number of other DMs to travel from A to B during rush hour [3]. Each edge is characterized by a certain travel time, depending on the traffic on this edge.

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2 The Problem Example

For concreteness, let us assume that 4000 cars (corresponding DMs) get from A to B by routes 1, 2. The equilibrium traffic is 2000 cars on each route with a travel time of 65 minutes. Let there be node C on route 1 and node D on route 2 , and the trip on the subroute $C-B$ takes 45 minutes, as does the trip on the subroute $A-D$ these subroutes are insensitive to congestion, and the travel time on each such subroute does not depend on the number of cars traveling on it.

On the other hand, suppose that the travel time on subroute $A-C$ or subroute $D-B$ is $x(A-C)/100$ minutes and $x(D-B)/100$ minutes, respectively, where $x(A-C)$ and $x(D-B)$ are the numbers of cars traveling on subroute $A-C$ and $D-B$, respectively. Then at $x(A-C) = 2000 = x(D-B)$ the total travel time along the route $A-C-B$ is 65 minutes. If $x(A-C) = 4000$, then the total travel time along the route $A-C-B$ is 85 minutes; similarly, the total travel time on the route $A-D-B$ is 85 minutes at $x(D-B) = 4000$.

In the language of game theory, here the DMs (drivers) are players, and the strategy of each of them is to choose a route $A-C-B$ or $A-D-B$.

We can expect the formation of equilibrium traffic, say, a Nash equilibrium (a list of 4000 strategies, where the strategy of each player $i = 1, \ldots, 4000$ is the best response of that player to the listed strategies of all other players $1, \ldots, i-1, i+1, \ldots, 4000$).

All Nash equilibria have an equal balance because at $x(A-C) > 2000$, $x(A-D) = 4000 - x(A-C) < 2000$ the total travel time along the route $A-C-B$ exceeds the total travel time along the route $A-D-B$, and therefore, the driver on the route $A-C-B$ has an incentive to switch to another route $A-D-B$.

The updated network corresponds to another unique Nash equilibrium with a travel time of 80 minutes from A to B.

In the state of equilibrium, each driver uses a route $A-C-D-B$ corresponding to the total travel time 80 minutes at $x(A-C) = 4000 = x(D-B)$: if the driver uses $A-C-B$ instead of $A-C-D-B$, the

total travel time is 85 (minutes); if the driver instead of $A-C-D-B$ uses $A-D-B$, then the total trip time is equal to 85 (minutes). The creation of edge $C-D$ mentioned effectively made the corresponding subroute $C-D$ the dominant strategy for each driver.

3 The Braess Paradox

Dietrich Braess first demonstrated an example where increasing the resources of a transport network does not guarantee an increase in its equilibrium performance, and with certain combinations of parameters it reduces its performance [4]. This phenomenon is called the Braess paradox.

In the above example, before the network update, the equilibrium travel time of route $A-C-B$ or $A-D-B$ is equal to 65 minutes, and after the update, the equilibrium travel time of route $A-C-D-B$ is equal to 80 minutes, then $(80-65)/65 < 1/3$. If before the network update, the equilibrium travel time along the route $A-C-B$ or $A-D-B$ is 60 minutes (a subroute $C-B$ trip takes 40 minutes, as well as a subroute $A-D$ trip), and after the update, the equilibrium travel time along the route $A-C-D-B$ is 80 minutes, then $(80-60)/60 = 20/60 = 1/3$.

A traffic pattern is the choice of a path by each driver, and the social cost of a given traffic pattern is the total travel time of all drivers required for using this pattern. Let there be 4 players, travel time by $C-D$ is 0, by $C-B$ or $A-D$ is 5, by $A-C$ or $D-B$ is x. Then there is scheme 1, where all players use the route $A-C-D-B$ (each player needs to travel 8 (minutes)), and there is scheme 2, where 2 players use the route $A-C-B$ (each player needs to travel $x+5=2+5=7$ (minutes)) and 2 players use the route $A-D-B$ (each player needs for the trip 7 (minutes)). The social cost of scheme 1 (single Nash equilibrium) is $4 \times 8 = 32$, and the social cost of scheme 2 (socially optimal scheme) is $4 \times 7 = 28$.

4 Conclusion

Any common network update should correspond to a certain measurable criterion of social gain, which each person understands in his or her own way. The reconfiguration of any network involves a certain coordination of all its participants.

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