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Numerical method for calculating electrical power in a multiwire transmission line

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Abstract

Transmission line equations are considered for a system consisting of an arbitrary number of electrical conductors. The numerical technique based on finite difference approximation is proposed. First, the original system of differential equations is written in Riemann invariants, and then the resulting equations are approximated according to the finite difference method. In order to obtain the final finite difference scheme with minimal numerical dispersion and dissipation the method of first differential approximation is applied. The novelty lies in obtaining a generalization to the case of an arbitrary number of conductors.

Keywords: transmission line equations, multiwire line, finite difference method, Riemann invariants.

1 Introduction

Interest in the theory of multiple conductor (multiwire), parallel transmission lines extends over the last years because of their numerous applications. The term multiwire transmission line (MTL) typically refers to a set of parallel conductors that serve to transmit electrical signals between two or more points, for example, a source and a load. With some exception, most of the published works has focused on the theory of two parallel, mutually coupled transmission lines. The transmission line equations for a system consisting of an arbitrary number of conductors are derived in [1, 2], starting with Maxwell's equations.

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Various methods for solving the transmission line equations for multiwire lines are examined in [2, 3]. The most commonly used numerical techniques for such problems are the finite element method, the finite difference time-domain (FDTD), or the transmission line matrix method [3]. Time-domain differential methods are becoming increasingly popular among the electromagnetic community because of their versatility and their ability to provide simulation results that are intuitively meaningful to circuit designers and microwave engineers. In particular, the FDTD technique offers a mathematically straightforward analysis method, suitable for arbitrary electromagnetic geometries. However, FDTD scheme [3] is sensitive to numerical dispersion and leads to strong non-physical oscillations in numerical solutions in the cases of short circuit and idling. The purpose of the present paper is to obtain reasonably accurate numerical techniques for solving MTL equations with minimal numerical dispersion and dissipation.

2 Mathematical model

Consider the propagation of electromagnetic energy through multiwire three-phase high-voltage transmission line with arbitrary number of conductors. The mathematical formulation of the problem represents the system of partial differential equations known as transmission line equations. The equations are derived from Maxwell equations and for unknown voltage vector $\boldsymbol{u}(x,t)$ and current vector $\boldsymbol{i}(x,t)$ have the following form

$$\boldsymbol{L}\frac{\partial \boldsymbol{i}}{\partial t} + \frac{\partial \boldsymbol{u}}{\partial x} + \boldsymbol{R}\boldsymbol{i} = 0 \tag{1}$$

$$\boldsymbol{C}\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \boldsymbol{i}}{\partial x} + \boldsymbol{G}\boldsymbol{u} = 0 \tag{2}$$

The domain of the solution of the problem (as well as the domain of the definition for unknown vector functions of the current and voltage) is the rectangle $D = [(x,t) : x \in (0,l), t \in (0,T_{max})]$, where l is the

length of the transmission line, T_{max} is the maximal time of calculating for $\boldsymbol{u}(x,t)$ and $\boldsymbol{i}(x,t)$. In an n-wire line the vector functions $\boldsymbol{u}(x,t)$ and $\boldsymbol{i}(x,t)$ have n components each, but $\boldsymbol{L}$, $\boldsymbol{C}$, $\boldsymbol{R}$, $\boldsymbol{G}$ in Eqs. (1) and (2) are symmetrical matrices $(n \times n)$ of linear inductances, capacitances, wire resistances and conductivities of insulation (vector objects are marked in bold). So we have a system of hyperbolic partial differential Eqs. (1) and (2). To obtain a unique solution, we must add to these equations the initial (when $t = 0$) and boundary (when $x = 0$ and $x = l$) conditions. We assume that at the initial time $t = 0$ there are no voltages and currents in the line

$$\boldsymbol{u}(x,0) = \boldsymbol{i}(x,t) = 0, x \in [0,l]. \tag{3}$$

At the input of the line, at $x = 0$, voltages are given, and at the output for $x = l$ we have a load with resistance $\boldsymbol{R}_s$

$$\boldsymbol{u}(0,t) = \boldsymbol{U}_0(t), \boldsymbol{u}(l,t) = \boldsymbol{R}_s \boldsymbol{i}(l,t). \tag{4}$$

After solving the formulated problem, the active power P is calculated as the average value for the period T of instantaneous power oscillations $p(x,t) = u(x,t)i(x,t)$

$$P(x,t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} u(x,\tau) i(x,\tau) d\tau.$$

Reactive power in the line Q is calculated by the formula

$$Q(x,t) = \frac{1}{\omega T} \int_{t-T/2}^{t+T/2} u(x,\tau) \frac{di(x,\tau)}{d\tau} d\tau =$$

$$= -\frac{1}{\omega T} \int_{t-T/2}^{t+T/2} i(x,\tau) \frac{du(x,\tau)}{d\tau} d\tau.$$

3 Numerical method

In order to solve numerically the formulated problem we propose to apply the finite difference method as follows. To construct the difference

scheme, we modify the initial system of Eqs. (1) and (2) and reduce it to a diagonal form, using Riemann invariants. For the transformed system, taking into account the initial and boundary conditions Eqs. (3) and (4), we construct a difference scheme possessing the properties of approximation and stability, and, hence, the convergence. Then using the method of the first differential approximation, we demonstrate that constructed difference scheme has the minimum possible values of dissipation and dispersion terms.

4 Conclusion

The mathematical formulation of the problem for a multiwire power transmission line is studied. The finite difference scheme with minimal values of dissipative and dispersion effects is constructed.

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