Proceedings of the 4th Conference of Mathematical Society of Moldova
CMSM4'2017, June 28-July 2, 2017, Chisinau, Republic of Moldova

Parallel algorithm to solve the bimatriceal subgames generated by the informational extended strategies

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Abstract

Parallel algorithm for mixed system with shared and distributed memory to solve bimatrix game generated by the informational extended strategies is described.

Keywords: computer science, game teory, parallel system, parallel algorithm.

According to [1] we can construct the normal form of the bimatrix game

$$\Gamma\left(\theta_1^\alpha, \theta_2^\beta\right) = \left\langle I, J, A^\alpha, B^\beta \right\rangle,$$

where $A^\alpha = ||a_{i_j^\alpha j_i^\beta}||_{i\in I}^{j\in J}$, $B^\beta = ||b_{i_j^\alpha j_i^\beta}||_{i\in I}^{j\in J}$, $i_j^\alpha \in I^\alpha$, $j_i^\beta \in J^\beta$ to be referred to as informational non-extended game generated by the informational extended strategies $\left(\theta_1^\alpha, \theta_2^\beta\right)$. The game $\Gamma\left(\theta_1^\alpha, \theta_2^\beta\right)$ is played as follows: independently and simultaneously each player $k = \overline{1,2}$ chooses the informational non-extended strategy $i \in I$, $j \in J$, after that the players 1 and 2 calculate the value of the informational extended strategies $i_j^\alpha = \theta_1^\alpha(j)$ and $j_i^\beta = \theta_2^\beta(i)$, after that each player calculates the payoff values $a_{i_j^\alpha j_i^\beta}$, $b_{i_j^\alpha j_i^\beta}$ and with this the game is finished. It is clear that for all strategy profiles (i, j) in the game $\Gamma = \langle A, B\rangle$ the following realization $\left(i_j^\alpha = \theta_1^\alpha(j), j_i^\beta = \theta_2^\beta(i)\right)$ in terms of the informational extended strategies will correspond.

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To determine the Nash equilibrium profiles in the bimatrix game of type $\Gamma\left(\theta_1^{\alpha}, \theta_2^{\beta}\right)$ we must do the following:

- using the "combinatorial algorithm" construct for all α, β the sets I^{α}, J^{β};
- for all fixed α, β construct the payoff matrices $A^{\alpha} = ||a_{i_j^{\alpha} j_i^{\beta}}||_{i \in I}^{j \in J}$, $B^{\beta} = ||b_{i_j^{\alpha} j_i^{\beta}}||_{i \in I}^{j \in J}$;
- using existent algorithms determine the set $NE\left(A^{\alpha}, B^{\beta}\right)$ of Nash equilibrium profiles in the bimatrix game with the matrices A^{α} and B^{β}.

The basic parallel strategy consists of three main steps. The **first step** is to partition the input into several partitions of almost equal sizes. The **second step** is to solve recursively the subproblem defined by each partition of the input. Note that these subproblems can be solved concurrently in the parallel system. The **third step** is to combine or merge the solutions of the different subproblems into a solution for the overall problem. The success of such strategy depends on whether or not we can perform the first and third steps efficiently [2]. To release the first step of the parallel strategy, that is to release data paralelization, we use the open source Scalable Linear Algebra PACKage (ScaLAPACK) [3]. Four basic steps are required to call a ScaLAPACK routine.

- Initialize the process grid.
- Distribute the matrix on the process grid.
- Call ScaLAPACK routine.

We describe the parallel algorithm to determine for all informational extended strategies of the player 1 $\theta_1^{\alpha} \in \Theta_1 = \{\theta_1^{\alpha} : J \to I\}_{\alpha=1}^{\varkappa_1}$ and respectively $\theta_2^{\beta} \in \Theta_2 = \left\{\theta_2^{\beta} : I \to J\right\}_{\beta=1}^{\varkappa_2}$ of the player 2, the Nash

equilibrium profiles of the informational non-extended bimatrix games

$$\begin{aligned}\Gamma\left(\theta_1^{\alpha},\theta_2^{\beta}\right) &= \left\langle I,J,A^{\alpha},B^{\beta}\right\rangle, A^{\alpha}=||a_{i_j^{\alpha}j_i^{\beta}}||_{i\in I}^{j\in J},\\ B^{\beta} &= ||b_{i_j^{\alpha}j_i^{\beta}}||_{i\in I}^{j\in J}, i_j^{\alpha}\in I^{\alpha}, j_i^{\beta}\in J^{\beta}\end{aligned}$$

generated by the informational extended strategies $\left(\theta_1^{\alpha},\theta_2^{\beta}\right)$. For conveniences we introduce the following notations $\widehat{a}_{ij}\equiv a_{i_j^{\alpha}j_i^{\beta}}$ and $\widehat{b}_{ij}\equiv b_{i_j^{\alpha}j_i^{\beta}}$ for all $i\in I$, $j\in J$. So the bimatrix game $\Gamma\left(\theta_1^{\alpha},\theta_2^{\beta}\right)=\left\langle I,J,A^{\alpha},B^{\beta}\right\rangle$ is equivalent to the following bimatrix game $\widehat{\Gamma}\left(\theta_1^{\alpha},\theta_2^{\beta}\right)=\left\langle I,J,\widehat{A}^{\alpha},\widehat{B}^{\beta}\right\rangle$, where $\widehat{A}^{\alpha}=\|\widehat{a}_{ij}\|_{i\in I}^{j\in J}$, $\widehat{B}^{\beta}=\left\|\widehat{b}_{ij}\right\|_{i\in I}^{j\in J}$. Denote by $NE\left[\widehat{\Gamma}\left(\theta_1^{\alpha},\theta_2^{\beta}\right)\right]$ the set of all Nash strategy profiles in the game $\widehat{\Gamma}\left(\theta_1^{\alpha},\theta_2^{\beta}\right)$.

The parallel algorithm to find the set of all equilibrium profiles

$$((i^*(\alpha,\beta),j^*(\alpha,\beta))\in NE\left[\widehat{\Gamma}\left(\theta_1^{\alpha},\theta_2^{\beta}\right)\right]$$

for all fixed $\alpha=\overline{1,\varkappa_1}$ and $\beta=\overline{1,\varkappa_1}$ consists of the following steps.

1. Using the MPI programming model and open source library ScaLAPACK-BLACS, initialize the processes grid $\{(\alpha,\beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$, and the root MPI process broadcasts to all $(\alpha,\beta)-$MPI processes the initial matrices $A=||a_{ij}||_{i\in I}^{j\in J}$, and $B=||b_{ij}||_{i\in I}^{j\in J}$ of the bimatrix game $\Gamma=\langle A,B\rangle$.
2. For all fixed type-players α and β all fixed MPI processes (α,β) using the OpenMP directives and combinatorial algorithm construct the sets $\mathcal{I}^{\alpha},\mathcal{J}^{\beta}$ (i.e. construct the informational extended strategies $.\theta_1^{\alpha},\theta_2^{\beta}$) whereupon the matrices $\widehat{A}^{\alpha}=\|\widehat{a}_{ij}\|_{i\in I}^{j\in J}$, $\widehat{B}^{\beta}=\left\|\widehat{b}_{ij}\right\|_{i\in I}^{j\in J}$, where $\widehat{a}_{ij}=a_{i_j^{\alpha}j_i^{\beta}}$, $\widehat{b}_{ij}=b_{i_j^{\alpha}j_i^{\beta}}$ and $i_j^{\alpha}\in I^{\alpha}, j_i^{\beta}\in J^{\beta}$.
3. All fixed MPI processes (α,β) using the OpenMP functions and ScaLAPACK routines eliminate from matrix $\widehat{A}^{\alpha}$ and $\widehat{B}^{\beta}$ the lines

that are strictly dominated in matrix $\widehat{A}^{\alpha}$ and columns that are strictly dominated in matrix $\widehat{B}^{\beta}$. Finally we obtain the matrices $\left(\widetilde{A}^{\alpha},\, \widetilde{B}^{\beta}\right)$, where $\widetilde{A}^{\alpha} = ||\widetilde{a}_{ij}||_{i\in\widetilde{I}}^{j\in\widetilde{J}}$, $\widetilde{B}^{\beta} = ||\widetilde{b}_{ij}||_{i\in\widetilde{I}}^{j\in\widetilde{J}}$ and cardinals $\left|\widetilde{I}\right| \leq |I|\,,\ \left|\widetilde{J}\right| \leq |J|\,.$

4. All fixed MPI processes (α, β) using the OpenMP functions, ScaLAPACK routines and existing algorithm determine all Nash equilibrium strategy profile of the bimatrix game with matrices $\widetilde{A}^{\alpha}$, $\widetilde{B}^{\beta}$ and construct the set $NE\left[\widehat{\Gamma}\left(\theta_1^{\alpha}, \theta_2^{\beta}\right)\right]$ for bimatrix game with matrices $\widehat{A}^{\alpha}$ and $\widehat{B}^{\beta}$.
5. Using ScaLAPACK-BLACS routines, the root MPI process gathers from processes grid $\{(\alpha, \beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$ the set $NE\left[\widehat{\Gamma}\left(\theta_1^{\alpha}, \theta_2^{\beta}\right)\right]$ of strategy profiles.

For this algorithm a C++ program has been developed using MPI functions, OpenMP directives and ScaLAPACK routines. Program has been testing on the control examples on the Moldova State University HPC cluster. The test results were consistent with theoretical results.

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