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The Synthesis Function Method for Solving the Multi-Criteria Linear-Fractional Model in Integers

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Abstract

In this paper, I will propose a method for solving the linear-fractional multi-criteria optimization model with identical denominators in whole numbers. The solving procedure involves assigning utilities (weights) to each criterion, after which a single-criterion linear-fractional optimization model is constructed and can be solved. By changing the utility values, we can determine a new optimal compromise solution(s). The algorithm is theoretically justified and it was tested successfully on several examples.

Keywords: Multi-criteria model in integers, efficient solution, optimal compromise solution.

1 Introduction

In many decision-making situations the optimal solution must be of the integer type. A number of studies can be listed here, such as: dynamic memory allocation, multiprocessor systems, general positioning problems. But when the problem is of multi-criteria non-liniar type, this effort increases considerably, but even more so when the solution must be of integer type. Currently there is an increased interest in such types of models, especially from a practical point of view.

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2 Problem formulation

The integer multi-criteria linear-fractional optimization problem is described as follows:

$$\begin{cases} \left\{ \begin{array}{c} \min \\ \max \\ x \in D \end{array} \right\} F_k(x) = \frac{\sum_{j=1}^{n} c_{kj} x_j}{\sum_{j=1}^{n} d_j x_j}, \quad k = \overline{1,r} \\ A \cdot x \leq b \\ x \in Z^+ \end{cases}, \tag{1}$$

in which: $D = \left\{ x = (x_1, x_2, \ldots, x_n)^T \,|Ax \leq b, x \in Z^+ \right\}$, $A = \|a_{ij}\|$ is an array of size $m \times n (m < n)$, $C = \|c_{kj}\|$, is an array of size $r \times n (r < n)$, d is a n-dimensional line vector. x is an n-dimensional column vector and b is an m-dimensional column vector.

In order to solve the model (1), we impose the next condition:

$$\sum_{j=1}^{n} d_j x_j \neq 0, \quad (\forall)\, x \in D.$$

3 Findings and theoretical justifications

The multi-criteria optimization models, as a rule, do not admit an optimal solution. The solution(s) is one that achieves the best compromise of all criteria, also called Pareto optimal, non-dominated, efficient [2]. We will present some ways of defining them for model (1).

1. The solution $x^* \in Z^+$ for the model (1) is the vector that optimizes a synthesis function of all r criteria, i.e.: $h(F) = h[F_1, F_2, ..., F_r]$.

2. The solution $x^* \in Z^+$ is the vector which minimizes a single criterion such as: $\phi(x^*) = \min_{x \in D} h(\Psi_1(x - X_1), ..., \Psi_1(x - X_r))$, in which $X_j = (x_{1j}, x_{2j}, ..., x_{nj})^T, j = \overline{1,r}$ is the optimal solution of each criterion, F_j, and Ψ_k is a distance type function between the vector $x \in D$ and optimal solution X_k for each criterion F_k.

3. The solution $x^* \in Z^+$ is a vector which belongs to a set of efficient solutions of integer type.

Definition 1. *The basic solution* $\overline{X} \in Z^+$ *of the model (1) is a basic efficient one if and only if for any other basic solution* $X \in Z^+$*, where* $X \neq \overline{X}$*, for which the relations* $F_{j_1}(X) \geq F_{j_1}(\overline{X})$ *are true, where* $j_1 \in (1, ..., j_2)$ *– indices corresponding to the maximum type of criteria, there immediately exists at least one index* $\exists j_l \in (j_2 + 1, ..., r)$*, of the minimum type for which is true the relation:* $F_{j_l}(X) > F_{j_1}(\overline{X})$ *or, if the relation* $F_{j_l}(X) \leq F_{j_1}(\overline{X})$ *is true for all indices corresponding to the minimum type of criteria which are* $j_l \in (j_2 + 1, ..., r)$*, immediately exists at least one index from the set of indices of the maximum type of criteria* $\exists j_1 \in (1, ..., j_2)$*, for which the next relation* $F_{j_2}(\overline{X}) < F_{j_2}(X)$ *is true.*

4 The Combinatorial synthesis algorithm

One of the most important problems that arises when solving the multi-criteria optimization problem in integers is: what kind of optimal solutions of each criterion we will use to build the synthesis function of all criteria, these being in R^+ or in Z^+? In order to answer to this question, we propose to apply the next combinatorial method.

Stage I.

1. At this stage, it is necessary to solve $2r$ unicriteria linear fractional programming problem from model (1), of which r are of the type: $F_j = optim_{x \in D} F_j(x)$ and the other r – of the type: $F_j^p = pessim_{x \in D} F_j(x)$ on the same admissible domain:

$$D = \{x \in R | Ax \leq b, x \geq 0\};$$

2. Next, we will analogically solve $2r$ linear fractional programming problems of integer type [1].

3. We will build the vectors of records of the optimal and of the worst values of the objective functions, using in each combinatorial vector both values of some criteria in Z^+ and of others in R^+.

Stage II.

1. Randomly considering one of the vector records of the optimal values of the objective functions and correspondingly of the worst values, we construct the synthesis function of the model [2], which ex-

presses the summary utility of all criteria thus: $G = \sum_{j=1}^{r} (\alpha_j F_j + \beta_j)$, We will maximize this function on the same admissible domain, using the utility maximization method [2]. The obtained optimal solution is one of the best compromise for model (1).

Theorem 1. *For any utility values assigned to the objective functions in model (1), where the identical denominator is nonzero over the admissible domain, the optimal compromise solution(s) corresponding to them remains the same for any combinatorial selection of the optimal and its corresponding pessimistic values of the criteria in R^+ or in Z^+.*

5 Conclusion

The proposed method is valuable in order to determine the optimal compromise solution(s) in intege of the model (1), because it allows combinatorial selection of the values of each objective function in R^+ or in Z^+. Last but not least, the decision maker has the free choce of celecting utility values criteria. This fact makes it possible to solve model interactively, increasing both efficiency and its attractiveness.

References

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