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Input-Output model for Republic of Moldova

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Abstract

The twenty three aggregated brunches of the Moldovan economy are considered. Squared 23x23 direct expenditure coefficients matrix was constructed. Optimization problem dealing with price energy growing impact on the final demand (Gross Domestic Product) has been formulated and solved.

Keywords: input-output table, optimization model, pure industry, final demand, direct costs coefficients matrix.

1 Introduction

W. W. Leontief [1] propounded the scheme of an input-output model. Idea of input-output coefficients (direct yield costs) of a single production industry for other production industries, suggested and implemented by W. W. Leontief is the ground of contemporary input-output model. These models were improved theoretically and practically: transiting from static model to dynamic one with the capital investments lag of one or more years, considering environmental factor, analysing and estimating the world economy's further development, forecasting and forming long-term and medium-term indicative planning of national economy. In this paper optimization model based on the input-output square matrix coefficients is considered. Square 23x23 matrix of input-output coefficients of the Moldovan economy in the period 1996-2014 was constructed. The problem of higher energy prices impact on the final demand has been formulated and solved.

2 Problem statement

The square part of the input-output table has n rows and n columns, and the figure in the i row and j column represents the amount of product from industry i delivered to industry j in a particular calendar

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year. The result of dividing that quantity by the total output of industry j is a coefficient measuring input per unit of output. In this way the $n \times n$ part of the flow table is converted to an $n \times n$ matrix of coefficients, where the entries in the j column include (when supplemented by the j column of factor inputs per unit of output) all inputs needed to produce one unit of output of industry j. This column of coefficients represents the average technology in use in industry j. For simplicity it is assumed that every industry is pure, so that a single characteristic output is produced using a single average technology. Denote A – the $n \times n$ matrix of interindustry coefficients , x – the $n \times 1$ vector of outputs, y – the likewise $n \times 1$ vector of final deliveries, while F is the $k \times n$ matrix of factor inputs. Mathematical models in input-output economics are reffered to per unit of output (one row for each of k factors) and total factor use is the vector f. Then the basic static input-output model is the following:

$$(I - A)^{-1} y = x, \tag{1}$$

$$f = Fx, \tag{2}$$

where $(I - A)^{-1}$ is the inverse matrix, so called the Leontief inverse. It is also known as the multiplier matrix, because the economy needs to produce a larger amount of a specific good (the amount of final demand for that good), final demand y needs to be multiplied by the obtained x. Equations (1) - (2) comprise the basic static input-output model. Much attention is given to conditions that guarantee the multiplier matrix is strictly positive. Such conditions make sense because basic economic logic requires that an increase $\Delta y > 0$ in final demand in equation (1) results in an increase $\Delta x > 0$ in total output. If the matrix $(I - A)^{-1}$ is not strictly positive, this logic could be violated, i.e. equation (1) always has a solution $x > 0$ for $y > 0$. In fact, the study of equation (1) has led to a number of equivalent statements about A, such as:

1. $(I - A)^{-1} > 0$.
2. $(I - A)^{-1} = I + A + A^2 + A^3 + \dots$, that is, the series $\sum A^k$ is convergent.
3. All successive principal minors of $(I - A)^{-1}$ are positive.

4. There exists a choice of units such as all row sums or all column sums of A are smaller than 1.
5. A has a dominant eigenvalue λ, where $0 < \lambda < 1$.
6. The dominant eigenvalue λ of A gets larger, if one element of A is increased, and λ gets smaller, if one element of A is decreased.

Statement 2 says that output $x = y + Ay + A(Ay) + \ldots$. So the quantity y should be produced, plus Ay, which is the vector of input to produce y, etc. Statement 3 is the well-known Hawkins-Simon condition, which assures that each subsystem is productive, i.e., each subgroup of industries i, j, k, requires less input from the economic system than it produces in terms of outputs. According to statement 4, the Brauer-Solow condition, value added in each sector is positive in coefficient matrices derived from input-output tables in (nominal) money values. That is, units for measuring physical output are such as each one costs one monetary unit (thus, if the output unit is lei, the unit price is 1.0 by definition). Assuming that the matrix describes a viable economy, this property assures that if output is measured in any chosen physical units, there exists a set of prices such as each industry has a positive value added (i.e., revenue left to pay for factor inputs). The dominant eigenvalue λ is a measure of the size of intermediate outputs produced in the economy in relation to total production. In other words, λ indicates the net surplus of an economy in the sense that the larger λ is (within the statement 5), the smaller the net output will be. The surplus so defined can be consumed, invested for growth, devoted to environmental protection, etc. Statement 6 is useful for interpreting the role of technological change. Earlier interbranch models, based on the input-output tables for economy of the Republic of Moldova in the period 1998-2004, were examined in [2]. In present article, using the input-output model described above, we tried to solve the following problem. Republic of Moldova imported all kinds of electric and thermic energy therefore its economy is very sensitive to energy prices movement, because these prices have a great impact over all economy, especially on goods and services. The energy industry, being a monopoly, determines prices by itself, in such a manner affecting economy

as a whole. The principal goal of this research is to determine how prices in energy sector influence other industries. In this respect input-output table will be used to formulate optimization problem restricted by input-output constraints.

3 Optimization Input-Output model

Now, having elaborated input-output tables for 23 aggregated industries of Moldovan economy in the period 1996-2014, let's formulate the optimization model. Suppose, that energy prices grow at 1,5, i.e. energetic technological column growth on 1,5. How demand vector (i.e. Gross Domestic Product) will be modified in such circumstances? Given the vector of output x, the problem is to maximise $\sum y_i$ restricted by $(I - A)^{-1}y = x$ or

$$\max D = \sum y_i, \tag{3}$$

under the following restrictions:

$$(I - A)^{-1}y = x \tag{4}$$

This optimisation problem, with known output for 23 industries and input-output table for 2014 year, was solved using the Solver application. Calculations effectuated at different rate (0,5;1,0; 1,5; etc.) have demonstrated that final demand diminishes together with energy prices growing. And the conclusion is the following: growth in prices should be done cautiously.

References

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