Proceedings of the International Conference IMCS-60, MSU
October 10–13, 2024, Chisinau, Republic of Moldova

Specific General Properties of Discrete W_q-Symmetry Groups

Alexandru Lungu

Abstract

Groups of W_q-symmetry transformations of the bounded geometrical figures regularly weighted with "physical" tasks with orientation are studied in the present work. Some specific general properties of W_q-symmetry groups are briefly analyzed.

Keywords: groups, symmetry, quasi-homeomorphisms, crossed wreath products, $\overleftarrow{G}$-invariant subgroups.

1 Introduction

The generalized symmetry called W_q-symmetry was analyzed in [1], and the general theory of discrete groups of W_q-symmetry was elaborated and developed in [1-7]. In the case of W_q-symmetry, the geometrical component g of the transformation $g^{(w)} = wg$ operates both on points and on "indexes"-qualities, attributed to the points, just like in the case of $\overline{P}$-symmetry [8,9], while the supplementary substitution of "indexes" essentially depends on the choice of points just like in the case of W_p-symmetry [10,11]. In this work, some specific general features of groups of W_q-symmetry are analyzed.

2 On the description of the discrete groups of W_q-symmetry

Identifying the groups G and W with their isomorphic injections into $W \overset{\rightarrow}{\leftthreetimes} G = P \overset{\rightarrow}{\wr} G$ by the rules $g \mapsto w_o g$, where w_o is the unit of the

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group W, and $w \mapsto w1$, where 1 is the unit of the group G, give us the posibility to find the subgroup $H = G^{(W_q)} \cap G$ of classical symmetry and the subgroup $V = G^{(W_q)} \cap W = G^{(W_q)} \cap W'$ of W-identical transformations from $G^{(W_q)}$.

Let $G^{(W_q)}$ be a group of W_q-symmetry with generating group G, initial transitive group of substitutions P, (i.e. $W = \overline{\prod}_{g_i \in G} P^{g_i}$, where $P^{g_i} \cong P$), subset $W' = \{w | g^{(w)} \in G^{(W_q)}\}$, the kernel H_1 of accompanying homomorphism $\tau : G \longrightarrow AutW$, symmetry subgroup H and the subgroup V of W-identical transformations.

The group $G^{(W_q)}$ is called major, minor or V-middle if $w_0 < V = W' = W, w_0 = V < W' = W$ or $w_0 < V < W' = W$, respectively. If W' is a non-trivial subgroup of W, then the group $G^{(W_q)}$ is called W'-semi-major, W'-semi-minor or (W', V)-semi-middle according to the cases when $w_0 < V = W', w_0 = V < W'$ or $w_0 < V < W'$. If $W' \subset W$, but W' is not a group, the group $G^{(W_q)}$ is called W'-pseudo-minor or (W', V)-pseudo-middle when $w_0 = V \subset W'$ or $w_0 < V \subset W'$ [1,8,11].

3 On specific properties of discrete W_q-symmetry groups

Any finite discrete group $G^{(W_q)}$ of W_q-symmetry satisfies the following specific properties:

1. The group operation in $G^{(W_q)}$ has the form $w_i g_i \otimes w_j g_j = w_k g_k$, where $g_k = g_i g_j$, $w_k = w_i^{g_j} \overrightarrow{\tau_{g_i}}(w_j), w_i^{g_j}(g_s) = w_i(g_j g_s)$ and $\overrightarrow{\tau_{g_i}}(w_j) = g_i w_j g_i^{-1}$;

2. The mapping λ of the group $G^{(W_q)}$ onto the group G by the rule $\lambda[g^{(w)}] = g$ is a homomorphism;

3. $W' = \{w | g^{(w)} \in G^{(W_q)}\}$ is a subset of group W; moreover, $w_0 \subset W' \subset W$, where w_0 is the unit of group W and W' in general is not a group.

4. The groups $G^{(W_q)}$ of W_q-symmetry are subgroups of the crossed standard Cartesian wreath product of P with the group G, accompanied by two morphisms: a) the isomorphism $\varphi : G \to AutW$ by the

rule $\varphi(g) = \overleftarrow{g}$, where automorphism $\overleftarrow{g}$ acts on the w elements, through left g-shift of their components ($\overleftarrow{g}$: $w \mapsto w^g$ and $w^g(g_k) = w(gg_k)$) and b) the homomorphism $\tau : G \to AutW$ by the rule $\tau(g) = \overrightarrow{\tau_g}$ (where $\overrightarrow{\tau_g}(w) = gwg^{-1}$);

5. The group $G^{(W_q)}$ contains as subgroup a group $G_1^{(W_1)}$ of $\overline{P}$-symmetry, where $W_1 = DiagW \bigcap W'$, from the family with the generating group G_1 ($G_1 < G$) and the same $Ker\tau = H_1$;

6. The group $G^{(W_q)}$ with the classical symmetry subgroup H contains as subgroup a group $H_1^{(W_p)}$ of W_p-symmetry from the family with the generating group H_1 and the symmetry subgroup H', where $H' = H \bigcap H_1$.

7. $V^g = V$ for any transformation $g^{(w)}$ from $G^{(W_q)}$, i.e. V is a $\overleftarrow{G}$-invariant subgroup;

8. $\overrightarrow{\tau_{g_i}}(V) = (w_i^{(g_j)})^{-1} V w_i^{g_j}$ for transformations $g_i^{(w_i)}$ and $g_j^{(w_j)}$ from $G^{(W_q)}$.

9. Any equivalent groups of W_q-symmetry have: a) the same generating group, b) the same initial group P of substitutions, c) conjugated symmetry subgroups, d) conjugated subgroups of W-identical transformations, e) equivalent subgroups of $\overline{P}$-symmetry, i) equivalent subgroups of W_p-symmetry.

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Alexandru Lungu

Chisinau/State University of Moldova
Email: lungu.al@gmail.com