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MILP Model for Scheduling Jobs and Crews with Employee Workload Balancing and Job Lateness

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Abstract

This study aims to propose an original and flexible mixed integer linear programming (MILP) model for scheduling jobs and crews. The proposed model minimizes the costs of assigning jobs to crews, employee workload heterogeneity and job lateness.

Keywords: scheduling, crew, workload balancing, lateness, milp

1 Introduction

Scheduling jobs and crews is essential in manufacturing and the service industry. Effective jobs assignment to crews can significantly reduce costs and increase company productivity level.

Paper [1] presents an excellent comparative analysis of mixed integer programming models (MILP) for the classical job shop scheduling problem. Many studies have also been conducted in various areas concerning job scheduling. Here are a few examples. The bus crew scheduling problem is studied in [2]. The manpower scheduling problem in manufacturing is explored in [3]. Such studies propose MILP models and heuristic algorithms to solve the problem and obtain optimal or near-optimal solutions.

This paper presents an original MILP model for scheduling job assignments to crews. The objective function is to minimize the total

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costs of assigning jobs to crews, employee workload heterogeneity and job lateness.

Regarding the problem, we consider the following assumptions. Each job is assigned to exactly one crew at a time. Each crew can perform many jobs at once, so the number of crew employees participating in work can be at most the total number of crew employees. For each pair of jobs and crews, there is a set of work options associated with the number of crew employees allocated to perform the job. The earliest start and the latest admissible end times are given for each job. Job lateness is allowed, but it is penalized. For each job, some predecessor jobs can be defined. There is a cost of assigning a job to a crew.

2 MILP notations

We consider time discrete, and the entire planning horizon is divided into elementary time intervals of the same duration $d, d > 0$. The index of an elementary time interval is associated with its start time, and it is denoted by t. The start time of each elementary time interval is found by multiplying its time index by d.

Sets and parameters:

J - set of jobs

C - set of crews

$O_{j,c}$ - set of work options of crew c on job j. An option is associated with the number of employees of crew c allocated to perform job j

T - set of time interval indexes of planning horizon, $T = \{0, 1, ..., |T|-1\}$, where 0 and $|T|-1$ are the planning horizon's first and last indexes. Here, 0 and $|T| \cdot d$ are the planning horizon's start and end times.

T_c^{crew} - set of working time intervals of crew c, $T_c^{crew} \subseteq T$

e_j - the earliest admissible time index for starting work on job j

l_j - the latest admissible time index for ending work on job j

n_c^{crew} - total number of employees in crew c

$n_{j,c,o}^{work}$ - number of employees required to perform job j by crew c according to option o

$d^{all}_{j,c,o,t}$ - number of all (working and non-working) time intervals during which work on job j is performed by crew c according to option o when work starts at the time corresponding to time index t

$d^{work}_{j,c,o,t}$ - number of working time intervals during which work on job j is performed by crew c according to option o when work starts at the time corresponding to time index t

$p_{j,j'}$ - predecessor indicator. $p_{j,j'} = 1$ iff job j is an immediate predecessor of job j'

$g_{j,j'}$ - gap time between ending of job j and starting of job j'

$\alpha_{j,c}$ - cost of choosing crew c for performing job j. This cost may express the skill of crew c in performing job j

β_j - lateness cost of job j

γ - cost of employee workload heterogeneity

Decision variables:

$x_{j,c,o,t}$ - binary variable that is equal to 1 iff crew c performs work on job j according to option o and starting at the time corresponding to time index t

y_j - non-negative integer variable indicating lateness of job j

z^{low}, z^{high} - non-negative real variables indicating the lowest and, respectively, the highest employee workload

3 MILP model

The objective (1) of the model shown below is to minimize the costs of assigning jobs to crews, employee workload heterogeneity and job lateness. Constraints (2) ensure that a unique crew, work option (number of employees), and start time are chosen for each job in the final solution. Constraints (3) guarantee that the number of crew employees assigned to the job at any given time interval is at most equal to the total number of crew employees. Constraints (4) require that the start time of any job be greater than the completion time of all its predecessors plus a given gap time. Constraints (5) are used to calculate job lateness. Constraints (6) and (7) are used to calculate the highest and

the lowest workload per employee. Finally, constraints (8)-(10) define types of variables.

$$\sum_{j\in J}\sum_{c\in C}\alpha_{j,c}\cdot\sum_{o\in O_{j,c}}\sum_{\substack{t\in T_c^{crew}\\ e_j\le t\le|T|-d^{all}_{j,c,o,t}}} x_{j,c,o,t}+$$

$$+\sum_{j\in J}\beta_j\cdot y_j+\gamma\cdot(z^{high}-z^{low})\to min \quad (1)$$

$$\sum_{c\in C}\sum_{o\in O_{j,c}}\sum_{\substack{t\in T_c^{crew}\\ e_j\le t\le|T|-d^{all}_{j,c,o,t}}} x_{j,c,o,t}=1 \quad \forall j\in J \quad (2)$$

$$\sum_{j\in J}\sum_{o\in O_{j,c}} n^{work}_{j,c,o}\cdot\sum_{\substack{t'\in T_c^{crew}\\ t-d^{all}_{j,c,o,t'}+1\le t'\le t}} x_{j,c,o,t'}\le n_c^{crew} \quad \forall c\in C,\forall t\in T_c^{crew} \quad (3)$$

$$\sum_{c\in C}\sum_{o\in O_{j,c}}\sum_{\substack{t\in T_c^{crew}\\ e_j\le t\le|T|-d^{all}_{j,c,o,t}}} (t+d^{all}_{j,c,o,t})\cdot x_{j,c,o,t}+g_{j,j'}\le$$

$$\le\sum_{c\in C}\sum_{o\in O_{j',c}}\sum_{\substack{t\in T_c^{crew}\\ e_{j'}\le t\le|T|-d^{all}_{j',c,o,t}}} t\cdot x_{j',c,o,t} \quad \forall j,j'\in J, p_{j,j'}=1 \quad (4)$$

$$\sum_{c\in C}\sum_{o\in O_{j,c}}\sum_{\substack{t\in T_c^{crew}\\ e_j\le t\le|T|-d^{all}_{j,c,o,t}}} (t+d^{all}_{j,c,o,t})\cdot x_{j,c,o,t}-l_j-1\le y_j \quad \forall j\in J \quad (5)$$

$$\sum_{j\in J}\sum_{o\in O_{j,c}} d^{work}_{j,c,o}\cdot n^{work}_{j,c,o}\cdot\sum_{\substack{t\in T_c^{crew}\\ e_j\le t\le|T|-d^{all}_{j,c,o,t}}} x_{j,c,o,t}\le n_c^{crew}\cdot z^{high} \quad \forall c\in C \quad (6)$$

$$\sum_{j\in J}\sum_{o\in O_{j,c}} d^{work}_{j,c,o}\cdot n^{work}_{j,c,o}\cdot \sum_{\substack{t\in T^{crew}_c \\ e_j\le t\le |T|-d^{all}_{j,c,o,t}}} x_{j,c,o,t} \ge n^{crew}_c\cdot z^{low} \quad \forall c\in C \tag{7}$$

$$x_{j,c,o,t}\in\{0,1\} \quad \forall j\in J, \forall c\in C, \forall o\in O_{j,c}, \forall t\in T^{crew}_c \tag{8}$$

$$y_j\in\mathbb{N} \quad \forall j\in J \tag{9}$$

$$z^{high}, z^{low}\in\mathbb{R}_+ \tag{10}$$

4 Conclusion

We proposed a precise MILP model for flexible scheduling jobs and crews with employee workload balancing and job lateness. Modern commercial MILP solvers (such as CPLEX and Gurobi) can tackle the jobs and crews scheduling problem in case of small and medium-sized instances. The near-optimal heuristic procedure can be used for large instances of the problem. Our next task is to further develop the model by adding job earliness and extending relationships between jobs.

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